DEPARTMENT OF COMMERCE

TECHNOLOGIC PAPERS
OF THE
BUREAU OF STANDARDS

S. W. STRATTON, DIRECTOR

No. 218

[Part of Vol. 16]

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BY

A. H. STANG, Associate Physicist
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Bureau of Standards

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RESULTS OF SOME COMPRESSION TESTS OF STRUCTURAL STEEL ANGLES.

By A. H. Stang and L. R. Strickenberg.

ABSTRACT.

This article presents the results of compression tests of 170 structural angles, made at the Pittsburgh branch, Bureau of Standards. The object of the tests was to determine the ultimate compressive strength of angles fastened at the ends in such ways as would closely correspond to their connections in the construction of transmission towers. There was also tested a series of angles with square ends. An end fixation factor was found to represent satisfactorily the effect of different types of end connections. Using this fixation factor, the average values for large slenderness ratios were well represented by Euler's formula. The results obtained from shorter columns agreed with the experimental and theoretical results of Kármán. The effect of eccentric loading was most marked at the slenderness ratios indicated by Kármán's theory.

CONTENTS.

I. INTRODUCTION.

Compression tests of 170 standard rolled structural steel angles were made at the Pittsburgh laboratory of the Bureau of Standards during the spring of 1917. The specimens tested were all furnished by the tower department of the American Bridge Co., which cooperated in planning the investigation and in carrying out the tests. As the angles were intended for legs and lattice members in electrical transmission tower construction, the greater number were tested with bolted ends, the bolting imitating the riveting used in the construction of the towers. For comparison a number of angles were also tested with flat ends.

II. METHOD OF TESTING.

The specimens were all tested in a 600,000-pound Olsen testing machine. (Fig. 1 shows a general view of the testing machine with an angle under load.) In order to determine the deformation that took place in the angle as the load was applied, a special compressometer was used which was so located as to measure the shortening of the centroidal axis of the specimen. A view of the compressometer attached to a test specimen is shown in Figure 2.

The angles with square ends, having no bolts, were mounted directly between the base and the straining head of the testing machine. In order to test the bolted specimens, special fixtures of structural material were bolted to the base and straining head of the machine, and the specimens bolted to these, as shown in Figure 2, a view of the connection used for two bolts in one leg of the angle. A specimen with ends folded is shown in Fig. 3.

The dimensions of the angles are shown in Tables 1 to 7, inclusive, and in Figures 4 to 15 accompanying them. The physical and chemical properties of the material in the angles, obtained incompletely from the mill test reports, are given in Table 8.

TABLE 1.—Results of Compression Tests on Angles with Square Ends, No Bolts.

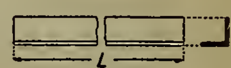

Fig. 4.

Specimen No.	Length L.	Size angle.	Maximum load (lbs./in.²) for slenderness ratio l/r=						
			50	100	150	200	250	300	350
	Ft. In.	Inches.							
A1	5 0	3 by 3 by 3/16		40,000					
A2	10 0	3 by 3 by 3/16				26,800			
A3	15 0	3 by 3 by 3/16						10,750	
A4	2 5½	3 by 3 by ¼	37,000						
A5	4 11	3 by 3 by ¼		36,000					
A6	7 4½	3 by 3 by ¼			32,500				
A7	9 10	3 by 3 by ¼				25,000			
A8	12 3½	3 by 3 by ¼					16,580		
A9	14 9	3 by 3 by ¼						10,750	
A10	17 2½	3 by 3 by ¼							9,000
A11	5 9	3½ by 3½ by ¼		33,000					
A12	11 6	3½ by 3½ by ¼				22,500			
A13	17 3	3½ by 3½ by ¼						12,660	
Average			37,000	36,300	32,500	24,800	16,580	11,390	9,000

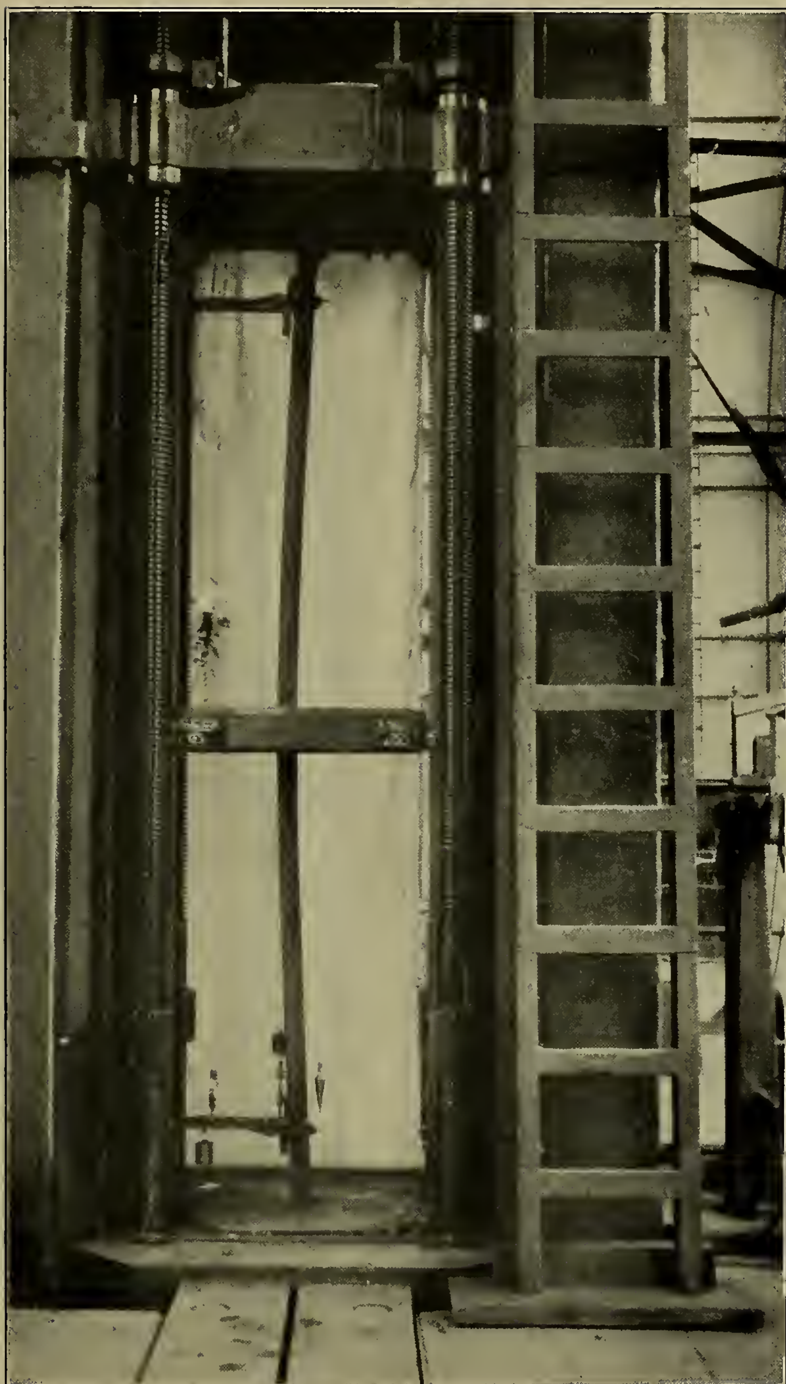

Fig. 1.—*View of the testing machine with an angle under load.*

Fig. 3.—*An angle with ends folded, in the testing machine.*

Fig. 2.—*View of compressometer and connections used for fastening a test specimen to the testing machine, two bolts in one leg.*

TABLE 2.—Results of Compression Tests on Angles with One Bolt Connection, Plain Ends.

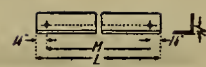

Fig. 5.

Specimen No.	Length L.	Distance. M.	Size angles.	Distance. N.	Diameter of holes.	Maximum load (lbs./in.^{2}) for slenderness ratio l/r= 200	250	300	350
	Ft. in.	Ft. in.	Inches.	Inches.	Inch.				
B1	4 4½	4 2	1¼ by 1¼ by ⅛	⅝	$\frac{9}{16}$	9,070	....	....	....
B2	6 5½	6 3	1¼ by 1¼ by ⅛	⅝	$\frac{9}{16}$	....	....	5,380	....
B6	4 2½	4 0	1¼ by 1¼ by $\frac{3}{16}$	⅝	$\frac{9}{16}$	10,710	....	....	....
B7	6 2½	6 0	1¼ by 1¼ by $\frac{3}{16}$	⅝	$\frac{9}{16}$	....	....	5,250	....
B8	5 2½	5 0	1½ by 1½ by ⅛	$\frac{13}{16}$	$\frac{11}{16}$	8,500	....	....	....
B9	6 5½	6 3	1½ by 1½ by ⅛	$\frac{13}{16}$	$\frac{11}{16}$	....	6,460	....	....
B10	7 8½	7 6	1½ by 1½ by ⅛	$\frac{13}{16}$	$\frac{11}{16}$	....	....	4,660	....
B11	5 0½	4 10	1½ by 1½ by $\frac{3}{16}$	$\frac{13}{16}$	$\frac{11}{16}$	10,150	....	....	....
B12	7 5½	7 3	1½ by 1½ by $\frac{3}{16}$	$\frac{13}{16}$	$\frac{11}{16}$	....	....	5,950	....
B13	8 6½	8 4	2 by 2 by ⅛	1	$\frac{11}{16}$	....	5,210	....	....
B14	10 2½	10 0	2 by 2 by ⅛	1	$\frac{11}{16}$	....	....	4,000	....
B17	8 6½	8 4	2 by 2 by $\frac{3}{16}$	1	$\frac{11}{16}$	....	5,500	....	....
B18	10 2½	10 0	2 by 2 by $\frac{3}{16}$	1	$\frac{11}{16}$	....	....	4,450	....
B19	10 7½	10 5	2½ by 2½ by ⅛	1¼	$\frac{11}{16}$	....	5,500	....	....
B20	12 8½	12 6	2½ by 2½ by ⅛	1¼	$\frac{11}{16}$	....	....	3,350	....
B21	14 9½	14 7	2½ by 2½ by ⅛	1¼	$\frac{11}{16}$	....	....	....	2,610
B22	12 5½	12 3	2½ by 2½ by $\frac{3}{16}$	1¼	$\frac{11}{16}$	....	....	4,000	....
B23	10 5	10 2½	2½ by 2½ by ¼	1¼	$\frac{11}{16}$	....	6,000	....	....
B24	12 5½	12 3	2½ by 2½ by ¼	1¼	$\frac{11}{16}$	....	....	4,760	....
B25	14 6	14 3½	2½ by 2½ by ¼	1¼	$\frac{11}{16}$	....	....	....	3,320
B26	12 8½	12 6	3 by 3 by ⅛	1½	$\frac{11}{16}$	....	4,670	....	....
B27	15 2½	15 0	3 by 3 by ⅛	1½	$\frac{11}{16}$	....	....	3,150	....
B28	17 8½	17 6	3 by 3 by ⅛	1½	$\frac{11}{16}$	....	....	....	2,210
B29	15 2½	15 0	3 by 3 by $\frac{3}{16}$	1½	$\frac{11}{16}$	....	....	3,890	....
B29A	15 2½	15 0	3 by 3 by $\frac{3}{16}$	1½	$\frac{11}{16}$	....	....	3,880	....
B30	12 6	12 3½	3 by 3 by ¼	1½	$\frac{11}{16}$	....	5,500	....	....
B30A	12 6	12 3½	3 by 3 by ¼	1½	$\frac{11}{16}$	....	5,400	....	....
BX30	12 6	12 3½	3 by 3 by ¼	1½	$\frac{11}{16}$	....	5,420	....	....
B31	14 11½	14 9	3 by 3 by ¼	1½	$\frac{11}{16}$	....	....	3,930	....
BX31	14 11½	14 9	3 by 3 by ¼	1½	$\frac{11}{16}$	....	....	4,450	....
B32	17 5	17 2½	3 by 3 by ¼	1½	$\frac{11}{16}$	....	....	....	2,750
BX32	17 5	17 2½	3 by 3 by ¼	1½	$\frac{11}{16}$	....	....	....	3,270
B33	17 5½	17 3	3½ by 3½ by $\frac{3}{16}$	1½	$\frac{11}{16}$	....	....	4,000	....
B34	20 4	20 1½	3½ by 3½ by $\frac{3}{16}$	1½	$\frac{11}{16}$	....	....	....	2,920
B35	17 5½	17 3	3½ by 3½ by ¼	1½	$\frac{11}{16}$	....	....	3,540	....
B36	20 4	20 1½	3½ by 3½ by ¼	1½	$\frac{11}{16}$	....	....	....	2,720
B37	11 5½	11 3	3½ by 2½ by ¼	1¼	$\frac{11}{16}$	....	5,000	....	....
B38	13 8½	13 6	3½ by 2½ by ¼	1¼	$\frac{11}{16}$	....	....	3,410	....
B39	15 11½	15 9	3½ by 2½ by ¼	1¼	$\frac{11}{16}$	....	....	....	3,500
Average	....	....	....	....	....	9,610	5,460	4,240	2,910

TABLE 3.—Results of Compression Tests on Angles with One Bolt Connection in One Leg Only, Ends Folded.

Fig. 6.

Specimen No.	Length L.	Distance M.	Size angle.	Distance N.	Diameter of holes.	Maximum load (lbs./in.^{2}) for slenderness ratio l/r= 100	200	300
	Ft. in.	Ft. in.	Inches.	Inch.	Inch.			
B3	2 3½	2 1	1¼ by 1¼ by ⅛	⅝	$\frac{9}{16}$	14,900	....	....
B4	4 4½	4 2	1¼ by 1¼ by ⅛	⅝	$\frac{9}{16}$	....	9250	....
B15	6 10½	6 8	2 by 2 by ⅛	1	$\frac{11}{16}$	....	7400	....
B16	10 2½	10 0	2 by 2 by ⅛	1	$\frac{11}{16}$	....	....	4500
Average	....	....	....	....	....	14,900	8325	4500

TABLE 4.—Results of Compression Tests on Angles with Two Bolt Connections, One Leg Only, Plain Ends.

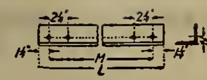

Fig. 7.

Specimen No.	Length L.	Distance. M.	Size angles.	Distance. N.	Diameter of holes.	Maximum load (lbs./in.²) for slenderness ratio 1/r=			
						100	150	200	250
	Ft. in.	Ft. in.	Inches.	Inches.	Inch.				
C1	2 5¾	1 10¾	1¼ by 1¼ by ⅛	⅝	9/16	27,200			
C2	2 4¾	1 9¾	1¼ by 1¼ by 3/16	⅝	9/16	24,200			
C3	2 10¾	2 3¾	1½ by 1½ by ⅛	13/16	11/16	23,500			
C3a	2 10¾	2 3¾	1½ by 1½ by ⅛	13/16	11/16	24,000			
C4	4 1¾	3 6¾	1½ by 1½ by ⅛	13/16	11/16		19,470		
C5	2 9¾	2 2¾	1½ by 1½ by 3/16	13/16	11/16	26,900			
C6	4 0¼	3 5¼	1½ by 1½ by 3/16	13/16	11/16		23,400		
C7	5 4¾	4 9¾	2 by 2 by ⅛	1	11/16		15,350		
C8	7 0¾	6 5¾	2 by 2 by ⅛	1	11/16			14,500	
C11	5 4¾	4 9¾	2 by 2 by 3/16	1	11/16		18,500		
C12	7 0¾	6 5¾	2 by 2 by 3/16	1	11/16			14,200	
C13	6 7¾	6 0¾	2½ by 2½ by ⅛	1¼	11/16		13,800		
C14	8 8¾	8 1¾	2½ by 2½ by ⅛	1¼	11/16			10,910	
C15	6 6¼	5 11¼	2½ by 2½ by 3/16	1¼	11/16		17,250		
C16	8 6¾	7 11¾	2½ by 2½ by 3/16	1¼	11/16			13,830	
C17	6 6¼	5 11¾	2½ by 2½ by ¼	1¼	11/16		16,995		
C18	8 6¾	7 11¾	2½ by 2½ by ¼	1¼	11/16			12,780	
C19	7 10¾	7 3¾	3 by 3 by ⅛	1½	11/16		12,400		
C20	10 4¾	9 9¾	3 by 3 by ⅛	1½	11/16			8,950	
C21	10 4¾	9 9¾	3 by 3 by 3/16	1½	11/16			12,520	
C21a	10 4¾	9 9¾	3 by 3 by 3/16	1½	11/16			13,400	
C22	7 9¼	7 2¼	3 by 3 by ¼	1½	11/16		15,210		
C22a	7 9¼	7 2¼	3 by 3 by ¼	1½	11/16		16,400		
C23	10 2¾	9 7¾	3 by 3 by ¼	1½	11/16			11,700	
C23a	10 2¾	9 7¾	3 by 3 by ¼	1½	11/16			12,530	
C24	10 2¾	9 7¾	3 by 3 by 5/16	1½	11/16			11,800	
C24a	10 2¾	9 7¾	3 by 3 by 5/16	1½	11/16			12,570	
C25	12 8¼	12 1¼	3 by 3 by 5/16	1½	11/16				10,000
C25a	12 8¼	12 1¼	3 by 3 by 5/16	1½	11/16				8,350
C26	11 10¾	11 3¾	3½ by 3½ by 3/16	1½	11/16			11,300	
C27	14 9¼	14 2¼	3½ by 3½ by 3/16	1½	11/16				8,350
C28	11 10¾	11 3¾	3½ by 3½ by ¼	1½	11/16			9,870	
C28a	11 10¾	11 3¾	3½ by 3½ by ¼	1½	11/16			10,400	
C29	14 9¼	14 2¼	3½ by 3½ by ¼	1½	11/16				6,300
C29a	14 9¼	14 2¼	3½ by 3½ by ¼	1½	11/16				7,850
C30	14 9¼	14 2¼	3½ by 3½ by 5/16	1½	11/16				8,800
C30a	14 9¼	14 2¼	3½ by 3½ by 5/16	1½	11/16				9,100
C31	9 4¾	8 9¾	3½ by 2½ by ¼	1½	11/16			11,800	
Average						25,200	16,880	12,050	8,400

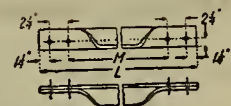

Fig. 8.

TABLE 5.—Results of Compression Tests on Angles with Two Bolt Connections, One Leg Only, Ends Folded.

Specimen No.	Length L.	Distance M.	Size angle.	Diameter of holes.	Maximum load (lbs./in.²) for slenderness ratio 1/r=	
					50	100
	Ft. in.	Ft. in.	Inches.	Inch.		
C9	2 0¾	1 5¾	2 by 2 by ⅛	$\frac{11}{16}$	16,530	
C10	3 8¾	3 1¾	2 by 2 by ⅛	$\frac{11}{16}$		18,000

TABLE 6.—Results of Compression Tests on Angles with Two Bolt Connections, One Bolt in Each Leg.

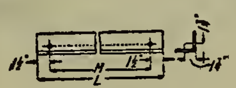

Fig. 9.

Specimens No. D1–D10.

Fig. 10.

Specimens No. E1–E8.

Specimen No.	Length L.	Distance M.	Size angle.	Diameter of holes.	Maximum load (lbs./in.²) for slenderness ratio 1/r=			
					200	250	300	350
	Ft. in.	Ft. in.	Inches.	Inch.				
D1	10 3	10 0	3 by 3 by $\frac{3}{16}$	$\frac{11}{16}$	13,820			
D2	12 9	12 6	3 by 3 by $\frac{3}{16}$	$\frac{11}{16}$		10,980		
D3	10 1	9 10	3 by 3 by ¼	$\frac{11}{16}$	11,620			
DX3	10 1	9 10	3 by 3 by ¼	$\frac{11}{16}$	12,400			
D4	12 6½	12 3½	3 ry 3 by ¼	$\frac{11}{16}$		9,760		
DX4	12 6½	12 3½	3 by 3 by ¼	$\frac{11}{16}$		8,500		
D5	10 1	9 10	3 by 3 by $\frac{5}{16}$	$\frac{11}{16}$	13,700			
DX5	10 1	9 10	3 by 3 by $\frac{5}{16}$	$\frac{11}{16}$	13,600			
D6	11 9	11 6	3½ by 3½ by ¼	$\frac{11}{16}$	9,420			
D7	14 7½	14 4½	3½ by 3½ by ¼	$\frac{11}{16}$		10,300		
D8	17 6	17 3	3½ by 3½ by ¼	$\frac{11}{16}$			6,520	
D9	14 7½	14 4½	3½ by 3½ by $\frac{5}{16}$	$\frac{11}{16}$		9,000		
D10	17 6	17 3	3½ by 3½ by $\frac{5}{16}$	$\frac{11}{16}$			6,130	
E1	16 8½	16 5½	4 by 4 by ¼	$\frac{11}{16}$		5,960		
E2	20 0	19 9	4 by 4 by ¼	$\frac{11}{16}$			4,530	
E3	23 3½	23 0½	4 by 4 by ¼	$\frac{11}{16}$				5,000
E4	20 0	19 9	4 by 4 by $\frac{5}{16}$	$\frac{11}{16}$			4,280	
E5	16 8½	16 5½	4 by 4 by ⅜	$\frac{11}{16}$		6,580		
E6	20 0	19 9	4 by 4 by ⅜	$\frac{11}{16}$			4,980	
E7	23 3½	23 0½	4 by 4 by ⅜	$\frac{11}{16}$				4,270
E8	19 9	19 6	4 by 4 by ½	$\frac{11}{16}$			3,930	
Average					12,400	8,720	5,060	4,630

TABLE 7.—Results of Compression Tests on Angles with Two or More Bolts in Each Leg.

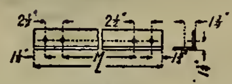

FIG. 11.

Specimens No. D11–D21.

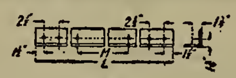

FIG. 12.

Specimens No. D22–D37.

FIG. 13.

Specimens No. E10–E12

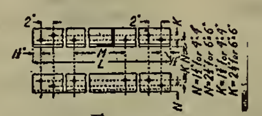

FIG. 14.

Specimens No. E14–E20.

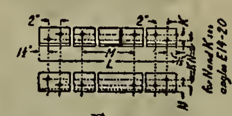

FIG. 15.

Specimens No. E21–E32.

Specimen No.	Length L.	Distance M.	Size angle.	Distance N.	Distance K.
	Ft. in.	Ft. in.	Inches.	Inches.	Inches.
D11	5 5½	4 9½	3 by 3 by 3/16	1¾	....
D12	7 11½	7 3½	3 by 3 by 3/16	1¾	....
D13	5 4½	4 8½	3 by 3 by ¼	1¾	....
D14	7 10	7 2	3 by 3 by ¼	1¾	....
D15	7 10	7 2	3 by 3 by 5/16	1¾	....
D16	10 1½	9 5½	3 by 3 by ⅜	1¾	....
D17	9 1	8 5	3½ by 3½ by ¼	1¾	....
D18	9 1	8 5	3½ by 3½ by 5/16	1¾	....
D19	11 11½	11 3½	3½ by 3½ by 5/16	1¾	....
D20	11 11½	11 3½	3½ by 3½ by ⅜	1¾	....
D21	11 9½	11 1½	3½ by 3½ by ½	1¾	....
D22	3 2	2 1	3 by 3 by 3/16	1¾	....
D23	3 1½	2 0½	3 by 3 by ¼	1¾	....
D24	3 4	1 10	3 by 3 by 5/16	1¾	....
D25	5 7	4 6	3 by 3 by 5/16	1¾	....
D26	3 6	1 7	3 by 3 by ⅜	1¾	....
D28	6 4	4 5	3 by 3 by ⅜	1¾	....
D28A	6 4	4 5	3 by 3 by ⅜	1¾	....
D29	3 9	2 3	3½ by 3½ by ¼	1¾	....
D30	6 5	5 4	3½ by 3½ by ¼	1¾	....
D31	3 11½	2 0½	3½ by 3½ by 5/16	1¾	....
D33	7 8	5 4	3½ by 3½ by 5/16	1¾	....
D33A	7 8	5 4	3½ by 3½ by 5/16	1¾	....
D34	4 2	1 10	3½ by 3½ by ⅜	1¾	....
D35	6 7½	5 1½	3½ by 3½ by ⅜	1¾	....
D35A	6 7½	5 1½	3½ by 3½ by ⅜	1¾	....
D36	4 4	1 7	3½ by 3½ by ½	1¾	....
D36A	4 4	1 7	3½ by 3½ by ½	1¾	....
D37	6 9	4 10	3½ by 3½ by ½	1¾	....
D37A	6 9	4 10	3½ by 3½ by ½	1¾	....
E10	13 7	13 0	4 by 4 by ¼	1½	1⅜
E11	13 7	13 0	4 by 4 by 5/16	1½	1⅜
E12	13 7	13 0	4 by 4 by ⅜	1½	1⅜
E14	7 2	6 3	4 by 4 by ¼	1½	1⅜
E15	4 2½	2 7½	4 by 4 by 5/16	1½	1⅜
E16	10 5½	9 6½	4 by 4 by ⅜	1½	1⅜
E17	7 5	5 10	4 by 4 by ½	1½	1⅜
E18	6 2½	3 11½	6 by 6 by ⅜	2½	2¼
E19	15 5½	14 6½	6 by 6 by ⅜	2½	2¼
E20	20 3	19 4	6 by 6 by ½	2½	2¼
E21	4 0½	2 9½	4 by 4 by ¼	1½	1⅜
E22	7 4	6 1	4 by 4 by 5/16	1½	1⅜
E23	4 4½	2 5½	4 by 4 by ⅜	1½	1⅜
E24	7 4	6 1	4 by 4 by ⅜	1½	1⅜
E25	4 8	2 1	4 by 4 by ½	1½	1⅜
E26	10 8	9 5	6 by 6 by ⅜	2½	2¼
E27	6 4	3 9	6 by 6 by ½	2½	2¼
E28	10 11	9 0	6 by 6 by ½	2½	2¼
E29	15 6	14 3	6 by 6 by ½	2½	2¼
E30	6 11½	3 0½	6 by 6 by ¾	2½	2¼
E30A	6 11½	3 0½	6 by 6 by ¾	2½	2¼
E31	11 2	8 7	6 by 6 by ¾	2½	2¼
E32	12 6	8 7	6 by 6 by ¾	2½	2¼

TABLE 7.—Results of Compression Tests on Angles with Two or More Bolts in Each Leg—Continued.

Specimen No.	Diameter of holes.	Number of holes in each leg.	Maximum load (lbs./in.²) for slenderness ratio l/r= 50	100	150	200
	Inch.					
D11	11/16	2	....	36,800	....	....
D12	11/16	2	....	....	26,050	....
D13	11/16	2	....	32,400	....	....
D14	11/16	2	....	....	29,900	....
D15	11/16	2	....	....	29,900	....
D16	11/16	2	....	....	....	22,700
D17	11/16	2	....	....	25,000	....
D18	11/16	2	....	....	25,000	....
D19	11/16	2	....	....	....	17,700
D20	11/16	2	....	....	....	19,900
D21	11/16	2	....	....	....	15,800
D22	11/16	3	41,000	....	....	....
D23	11/16	3	35,500	....	....	....
D24	11/16	4	36,000	....	....	....
D25	11/16	3	....	35,300	....	....
D26	11/16	5	35,500	....	....	....
D28	11/16	5	....	33,800	....	....
D28A	11/16	5	....	31,600	....	....
D29	11/16	4	35,000	....	....	....
D30	11/16	3	....	33,000	....	....
D31	11/16	5	36,000	....	....	....
D33	11/16	6	....	34,000	....	....
D33A	11/16	6	....	35,700	....	....
D34	11/16	6	38,000	....	....	....
D35	11/16	4	....	34,900	....	....
D35A	11/16	4	....	34,000	....	....
D36	11/16	7	41,800	....	....	....
D36A	11/16	7	41,200	....	....	....
D37	11/16	5	....	34,100	....	....
D37A	11/16	5	....	33,900	....	....
E10	11/16	2	....	....	....	14,250
E11	11/16	2	....	....	....	16,700
E12	11/16	2	....	....	....	20,150
E14	11/16	3	....	33,500	....	....
E15	11/16	5	37,000	....	....	....
E16	11/16	3	....	....	25,600	....
E17	11/16	5	....	30,000	....	....
E18	13/16	7	31,600	....	....	....
E19	13/16	3	....	....	22,800	....
E20	13/16	3	....	....	....	20,000
E21	11/16	4	37,500	....	....	....
E22	11/16	4	....	34,480	....	....
E23	11/16	6	36,200	....	....	....
E24	11/16	4	....	33,000	....	....
E25	11/16	8	31,000	....	....	....
E26	13/16	4	....	30,200	....	....
E27	13/16	8	28,300	....	....	....
E28	13/16	6	....	27,900	....	....
E29	13/16	4	....	....	27,750	....
E30	13/16	12	31,700	....	....	....
E30A	13/16	12	32,150	....	....	....
E31	11/16	8	....	35,000	....	....
E32	13/16	12	....	28,820	....	....

TABLE 8.—Results of Tests on Coupon Specimens.

Test numbers.	Size angle.	Chemical analysis (per cent).				Yield point.	Tensile strength.	Elongation in 8 inches.	Reduction of area.
		C.	Mn.	P.	S.				
	Inches.					Lbs./in.²	Lbs./in.²	Per ct.	Per ct.
B1, B2, B3, B4, C1	1¼ by 1¼ by ⅛								
B6, B7, C2	1¼ by 1¼ by 3/16	0.18	0.43	0.018	0.036				
B8, B9, B10, C3, C4	1½ by 1½ by ⅛	.20	.49	.016	.049				
B11, B12, C5, C6	1½ by 1½ by 3/16	.22	.57	.015	.043				
B13, B14, B15, B16, C7, C8, C9, C10.	2 by 2 by ⅛	.19	.49	.039	.045				
B17, B18, C11, C12	2 by 2 by 3/16	.17	.48	.014	.036				
B19, B20, B21, C13, C14	2½ by 2½ by ⅛			.016	.040	38,160	61,020	27.5	58.6
B22, C15, C16	2½ by 2½ by 3/16			.018	.034	38,480	60,640	28.7	56.0
B23, B24, B25, C17, C18	2½ by 2½ by ¼								
B26, B27, B28, C19, C20	3 by 3 by ⅛					46,680	58,730	28.2	57.4
A1, A2, A3, B29, C21, D1, D2, D11, D12, D22.	3 by 3 by 3/16	.22	.40	.016	.045	37,320	62,300	30.0	54.0
A4, A5, A6, A7, A8, A9, A10, B30, B31, B32, C22, C23, D3, D4, D13, D14, D23.	3 by 3 by ¼	.22	.41	.024	.042	36,710	58,020	28.7	55.7
C24, C25 D5, D15, D24, D25.	3 by 3 by 5/16	.18	.36	.036	.044	36,080	58,980	30.0	53.2
D16, D26, D28	3 by 3 by ⅜	.21	.34	.012	.040	36,840	65,540	27.5	49.6
B33, B34, C26, C27	3½ by 3½ by 3/16	.21	.36	.025	.040	38,000	57,930	28.7	52.8
A11, A12, A13, B35, B36, C28, C29, D6, D7, D8, D17, D29, D30.	3½ by 3½ by ¼								
C30, D9, D10, D18, D19, D31, D33.	3½ by 3½ by 3/16			.011		38,020	60,500	27.5	57.2
D20, D34, D35	3½ by 3½ by ⅜			.016		37,830	60,080	26.2	55.8
D21, D36, D37	3½ by 3½ by ½			.015		38,720	61,980	27.5	55.0
E1, E2, E3, E9, E10, E14, E21.	4 by 4 by ¼	.21	.44	.020	.037	35,850	58,600	26.2	52.2
E4, E11, E15, E22	4 by 4 by 5/16								
E5, E6, E7, E12, E16, E23, E24.	4 by 4 by ⅜								
E8, E17, E25	4 by 4 by ½	.25	.53	.013	.040	35,780	64,750	30.0	50.9
E13, E18, E19, E26	6 by 6 by ⅜	.21	.43	.014	.046	36,200	57,920	30.0	53.9
E20, E27, E28, E29	6 by 6 by ½	.24	.39	.013	.031	36,520	60,090	28.7	52.9
E30, E31, E32	6 by 6 by ¾	.22	.40	.032	.040	36,890	62,950	28.7	52.8
B37, B38, B39, C31	3½ by 2½ by ¼	.21	.35	.015	.030	36,870	60,780	27.5	52.1

TABLE 9.—Comparison of Lateral Deflection to Strength of Angles with One Bolt Connection.

Specimen No.	Slenderness ratio.	Lateral deflection at 4/9 S.	Rank. By strength.	Rank. By deflection.	Rank. Difference in—
	1/r	Inch.			
B1	200	0.11	3	2	1
B6	200	.06	1	1	0
B8	200	.17	4	4	0
B11	200	.16	2	3	1
B9	250	.10	1	1	0
B13	250	.20	8	4	4
B17	250	.23	3-5	5	0
B19	250	.19	3-5	2-3	0
B23	250	.19	2	2-3	0
B26	250	.33	10	10	0
B30	250	.25	3-5	6-7	1
B38	250	.31	7	8	1
BX30	250	.25	6	6-7	0
B37	250	.32	9	9	0
B2	300	.13	2	1	1
B7	300	.15	3	2-3	0
B10	300	.16	5	4	1
B12	300	.15	1	2-3	1
B14	300	.17	8-10	5	3
B18	300	.24	6-7	10-11	3
B20	300	.54	16	16	0
B22	300	.25	8-10	12	2
B24	300	.20	4	6-7	2
B27	300	.73	17	17	0
B29	300	.21	12	8	4
B29A	300	.24	13	10-11	2
B31	300	.32	11	13	2
BX31	300	.23	6-7	9	2
B33	300	.20	8-10	6-7	1
B35	300	.40	14	14	0
B38	300	.49	15	15	0
B21	350	.36	7	5	2
B25	350	.27	2	4	2
B28	350	.42	8	6	2
B32	350	.64	5	8	3
BX32	350	.15	3	1	2
B34	350	.21	4	3	1
B36	350	.50	6	7	1
B39	350	.20	1	2	1

TABLE 10.—Comparison of Lateral Deflection to Strength of Angles with Two Bolt Connections in One Leg Only.

Specimen No.	Slenderness ratio.	Lateral deflection at 4/9 S.	Rank. By strength.	Rank. By deflection.	Rank. Difference in—
	1/r	Inch.			
C4	150	0.12	2	2	0
C6	150	.10	1	1	0
C7	150	.23	7	5	2
C11	150	.17	3	3	0
C13	150	.32	9	9	0
C15	150	.25	4	6–7	2
C17	150	.20	5	4	1
C19	150	.23	10	10	0
C22	150	.27	8	8	0
C22A	150	.25	6	6–7	0
C8	200	.19	1	2–3	1
C12	200	.13	2	1	1
C14	200	.24	13	6–9	4
C16	200	.20	3	4	1
C18	200	.29	5	14	9
C20	200	.26	16	12	4
C21	200	.24	8	6–9	0
C21A	200	.24	4	6–9	2
C23	200	.26	11	11	0
C23A	200	.24	7	6–9	0
C24	200	.27	9–10	13	3
C24A	200	.23	6	5	1
C26	200	.25	12	10	2
C28	200	.35	15	16	1
C28A	200	.30	14	15	1
C31	200	.19	9–10	2–3	6
C25	250	.27	1	1	0
C25A	250	....	4–5	4	....
C27	250	.30	4–5	2	2
C29	250	.64	7	7	0
C29A	250	.32	6	5	1
C30	250	.35	3	6	3
C30A	250	.31	2	3	1

TABLE 11.—Comparison of Lateral Deflection to Strength of Angles with Two Bolt Connections, One Bolt in Each Leg.

Specimen No.	Slenderness ratio.	Lateral deflection at 4/9 S.	Rank. By strength.	Rank. By deflection.	Rank. Difference in—
	1/r	Inches.			
D1	200	0.10	1	4	3
D3	200	.24	5	5	0
DX3	200	.08	4	2	2
D5	200	.05	2	1	1
DX5	200	.09	3	3	0
D6	200	.47	6	6	0
D2	250	.10	1	1–4	0
D4	250	.18	3	5	2
DX4	250	.10	5	1–4	1
D7	250	.10	2	1–4	0
D9	250	.10	4	1–4	0
E1	250	.46	7	7	0
E5	250	.35	6	6	0
D8	300	.27	1	1	0
D10	300	.48	2	4	2
E2	300	.35	4	3	1
E4	300	.81	5	5	0
E6	300	.31	3	2	1
E8	300	1.05	6	6	0
E3	350	.22	1	1	0
E7	350	.35	2	2	0

TABLE 12.—Comparison of Lateral Deflection to Strength of Angles with Two or More Bolts in Each Leg.

Specimen No.	Slenderness ratio.	Lateral deflection at 4/9 S.	Rank.		
			By strength.	By deflection.	Difference in—
	l/r	Inch.			
D11	100	0.03	1	2	0
D13	100	.05	15	5-14	1
D25	100	.05	3	5-14	2
D28	100	.09	11	17	6
D28A	100	.05	16	5-14	2
D30	100	.05	13	5-14	0
D33	100	.05	8	5-14	0
D33A	100	.04	2	3	1
D35	100	.05	5	5-14	0
D35A	100	.05	9	5-14	0
D37	100	.05	7	5-14	0
D37A	100	.05	10	5-14	0
E14	100	.05	12	5-14	0
E17	100	.08	18	16	2
E22	100	.02	6	1	5
E24	100	.08	14	15	1
E26	100	.30	17	19	2
E28	100	....	20	20	....
E31	100	....	4	4	....
E32	100	.12	19	18	1
D12	150	.10	4	4-5	0
D14	150	.05	1-2	1	0
D15	150	.09	1-2	3	1
D17	150	.10	6-7	4-5	1
D18	150	.06	6-7	2	4
E16	150	.15	5	6	1
E19	150	.22	8	7-8	0
E29	150	.22	3	7-8	4
D16	200	.10	1	4	3
D19	200	.16	4	6	2
D20	200	.05	3	2	1
D21	200	.12	6	5	1
E10	200	.19	7	7	0
E11	200	.07	5	3	2
E12	200	.03	2	1	1

TABLE 13.—End Fixation Factors for Various End Connections of Angles.

End connection.	Fixation factor.	End connection.	Fixation factor.
Angles with square ends, no bolts	1.9	Two bolts, one in each leg	1.3
Two or more bolts in each leg	1.5	One bolt	1.1
Two bolts, in one leg only	1.3	Ends folded	1.1

III. RESULTS AND DISCUSSION OF TESTS.

(*a*) General Discussion.—The value of the maximum load sustained by each column was measured. These values are given in Tables 1 to 7, inclusive, and have been plotted against the values of the slenderness ratio l/r in Figures 16 and 17. In these figures the average value of the maximum loads for each slenderness ratio is shown by a solid circle. Full lines connect these average values. It will be noted from Figure 17 that for any given slenderness ratio the individual results are quite scattered, and when conclusions are drawn from the average values this fact must be kept in mind.

The manner in which the angles were held in the testing machine exerted a great influence on their strength. In other words, the strength of a column varies with the "degree of end fixation." The amount of this "end fixation" may be expressed by a fixation factor $f = l/L$, where l is the actual length of the member and L the length of the round end member which would fail under the same load; i. e., the "free length" of the member. Thus, the end fixation factor would be 1.0 for a column with round ends and 2.0 for a specimen tested with fixed ends.

The angles with square ends that were placed directly between the head and base of the testing machine would thus have an end

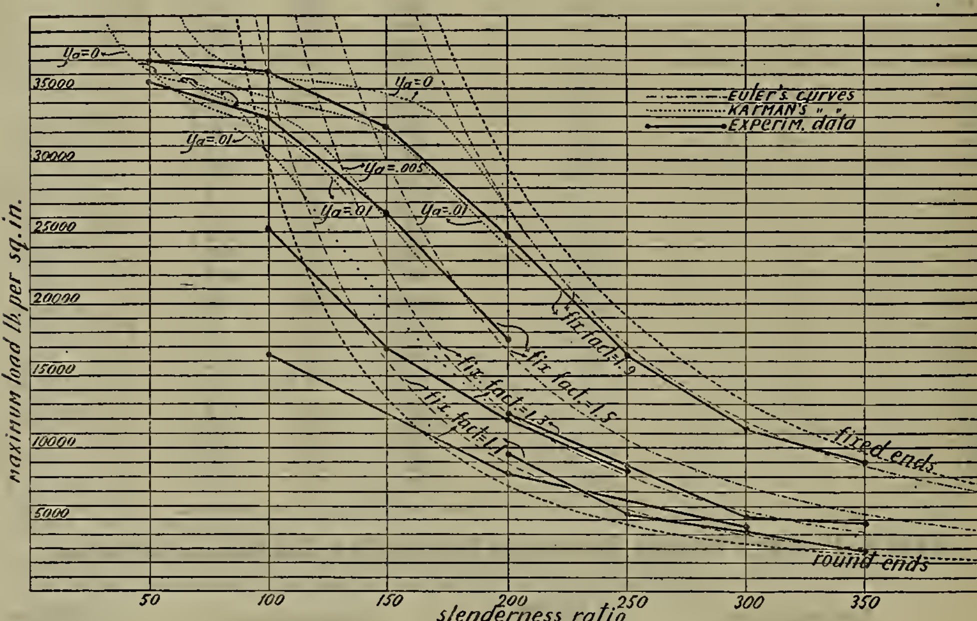

FIG. 16.—*Relation of maximum load to slenderness ratio.*

fixation of approximately the same degree as a column with theoretically fixed ends under axial loading, while specimens that were held with one bolt in one leg would be expected to approximate a round-end specimen under eccentric loading. When more than one bolt is used, the fixation factor would increase and approach 2.0 as the limit for the most rigidly held columns. End fixation factors for various end connections are given in Table 13.

It must be pointed out that in such column tests there is always present some eccentricity, due to imperfect centering in the testing machine and also to the manner in which the load is applied to the specimen, as by bolted connections. It is very difficult to accurately center even a short compression test specimen. The load was eccentric for all the angles bolted to their end con-

nections, and this eccentricity of loading always produces a diminution of the maximum load.

The results of the tests were compared with several types of column formulas. Formulas of the Rankine-Gordon type represent the results fairly well for values of the slenderness ratio up to about 150, but the longer column results are evidently best represented by the Euler formula:

$$\frac{P}{a}=\frac{\pi^2 E}{\left(\frac{l}{fr}\right)^2}$$

where P = total load, pounds.
a = cross-section area, square inches.
E = modulus of elasticity, pounds per square inch.
l = length of column, inches.
r = radius of gyration, inches.
and f = fixation factor.

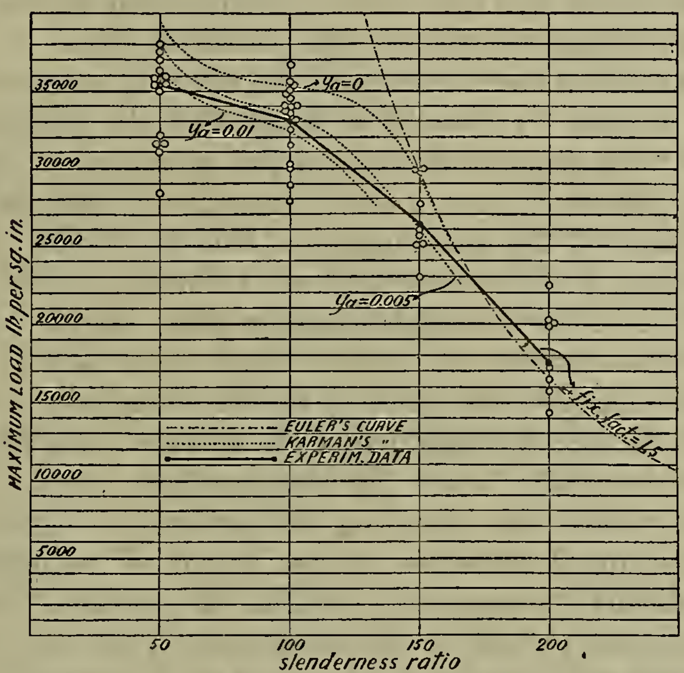

FIG. 17.—*Relation of maximum load to slenderness ratio for fixation factor f*=1.5.

The Euler formula does not give the strength of short columns, however, since for such lengths the elastic limit of the material is passed before the column fails. The theory of the deviation of short columns from Euler's law has been worked out by Considére (K. Considére, Résistance des Pièces Comprimées, Comptes Rendus, Congrès International des Procédés de Construction, pp. 371–397, 1891), Jasinsky (Jasinsky, Zu den Knickfragen, Schweiz. Bauzeitung, vol. 25, p. 172, 1895), and Kármán (Theo. von Kár-

mán, Untersuchung über Knickfestigkeit, Forschungsarbeiten a. d. Gebiete d. Ingenieurwesens, No. 81, 1910), and later independently by Southwell (R. V. Southwell, The Strength of Struts, Engineering, vol. 94, pp. 248–250, 1912; Aircraft Engineering, vol. 1, p. 20 et seq., January, 1920). The theoretical curve given by Kármán for a steel whose physical properties had been determined were recalculated for a yield point of 37,000 lbs./in.^{2} and a modulus of elasticity of 30,000,000 lbs./in.^{2} They were found to agree with the results of these tests when the end fixation factor was taken into account and the effect of eccentricity noted. This curve, which goes over into the Euler hyperbola for large values of the slenderness ratio, has been plotted for comparison with the average results of this test.

In Figures 16 and 17 the dashed curves represent the Euler formula for round and for fixed ends, as shown. The dash-and-dot curves shown represent the intermediate degree of end fixation for the Euler and their dotted continuation for the Kármán values that seemed to best fit the particular case.

Kármán made a special study of the effect of eccentric loading in column testing. It should be noted that the unit stresses he found are those from tests on 0.50 per cent carbon steel. The important feature is that a small eccentricity has the greatest effect in reducing the maximum unit load for values of slenderness ratio from 80 to 85 for round end columns ($f = 1.0$), and this is probably true for the milder steel in these angles. For other degrees of end fixation the critical slenderness ratio is obtained by multiplying, say, the value 85 by the value of f. It will be seen in the detailed discussion that the average value of the maximum unit load falls below the Kármán-Euler curves for these slenderness ratios, thus denoting the presence of eccentricity.

Since the lateral deflection at mid height of the columns was measured during the tests, it is possible to obtain a rough comparative measure at least of the eccentricity of the test specimens. For a column with fixed ends there can be no effective eccentric loading. The load, no matter how far its point of application is from the centroidal axis of the column, can only produce such stresses in a fixed end specimen as would be produced by a load concentrically applied. The reason for this is that the definition of a "fixed end" column presupposes that the tangent to the elastic curve at one end is parallel to and remains parallel to the tangent at the other end. In a testing machine, if the columns were really to have fixed ends, the bearing plates would remain

parallel to each other throughout the test, and all effects of the eccentric loading would be taken up by the supporting screws of the testing machine. As a matter of fact, however, it is impossible to maintain this theoretical condition of fixed ends under an eccentric load either in a testing machine or in a built-up structure, and the column strength will be reduced if the load is applied eccentrically.

From the elastic theory one may express the relation between the lateral deflection at mid height, y_m, and the initial eccentricity, y_a, for round end columns, as follows:

$$y_m = y_a \left(\frac{1}{\cos \frac{l}{2r}\sqrt{\frac{P^1}{aE}}} - 1 \right)$$

where P^1 is the load which produced the deflection y_m. Now, there is some value of $P^1 = kP$ (P being the value of the maximum load from Euler's formula) for which the lateral deflection y_m is equal to the initial eccentricity y_a. Solving for k under this condition, $k = 4/9$. That is, in a perfectly elastic round end column of any slenderness ratio the deflection at mid height when the load is 4/9 of the theoretical maximum is equal to the initial eccentricity of load. For other degrees of end fixation this ratio would be different, but for the sake of comparison Tables 9 to 12 show the lateral deflection at mid height which occurred at the unit loads $S^1 = 4/9\,S$. It is assumed that the value of the theoretical unit load S is given by the dash-and-dot curves of Figures 16 and 17. Any other definite ratio might have been chosen for the comparison, provided the ratio were small enough, but the comparative results would have been practically the same. No claim is made that these values represent the actual initial eccentricity. It is, however, evident that in practically all cases the specimen of given slenderness ratio and degree of end fixation which sustained the highest unit load also suffered the least lateral deflection at the unit load S^1, while the specimen which suffered the greatest lateral deflection sustained the least unit load. Tables 9 to 12 also give the "rank" of the specimens according to strength and to lateral deflection. With very few exceptions the rank of a specimen is practically the same by either method of ranking.

One might conclude, then, from these results that the theoretical load-slenderness ratio curve for zero eccentricity should be drawn somewhat above the largest load values, and thus obtain a different value of fixation factor from the value obtained by considering the mean of the test results. It must, however, be pointed out

that the specimens closely represented the conditions in actual construction, and no better centering of a member would be obtained on the average than was obtained in these tests. The mean results are therefore of more importance in design than any such theoretically determined values would be.

(*b*) DETAILED DISCUSSION OF RESULTS.—Figure 16 shows the results of tests of angles with square ends. The average result line is close to the fixed end curve (dotted), but agrees still better with the dash-and-dot curve plotted for $f = 1.9$. At $l/r = 200$ the average result is below the curve, and it is in this region that the most marked effects of eccentric loading are to be expected.

When the angles were tested with one bolt connection—in one leg only, approximately round end columns,—the results are close to the curve of end fixation, for $f = 1.1$. No appreciable effect of eccentricity in loading in reducing the maximum load appears here in the average results because the slenderness ratio is so much greater than 85.

The results of tests of angles with folded ends, do not fall so close to the curve for $f = 1.1$ as did those just considered. So few specimens of this class were tested that it is impossible to draw any definite conclusion whether this type of column curve is suitable for angles with ends folded. It must also be noted that angles with ends folded, as shown in Figures 3, 6, and 8, have a variable radius of gyration from section to section. The ordinary column formulas are not derived for such conditions.

Figure 16 shows also the results of the tests of specimens held at each end with two bolts, in one leg only. This manner of fastening is more rigid than when a single bolt is used and the results for the columns with slenderness ratio as large as 200 lie close to the Euler curve for $f = 1.3$. For shorter columns the average results lie below this curve, and this may be due to the eccentric loading which would have the greatest effect at $l/r = 110$.

When two bolts are used, one in each leg, the degree of end fixation appears to be the same as for the previously considered class, and the results fall very close to the Euler curve for $f = 1.3$. The lengths tested in these two classes overlap for the slenderness ratios 200 and 250, and the average results for each of these slenderness ratios are nearly equal. The strength of the angles held with two bolts, in one leg only, is apparently the same as for specimens held with two bolts, one in each leg.

When two or more bolts were used in each leg for fastening the angle to the testing machine, the end fixation factor is still

larger, and the curve for $f=1.5$ of the Kármán-Euler type represents the average results very well, as shown in Figures 16 and 17. Here, again, the eccentricity lowers the average result value at the critical slenderness ratio value, $85\times1.5=127.5$, and is visible at $l/r=150$. When angles are held as rigidly as these were, it might have been expected that the end fixation factor would have been closer to the fixed end condition, $f=2.0$. It may be that the factor is no higher than 1.5 because of the deformation which doubtless occurred in the structural members to which the test pieces were bolted.

IV. CONCLUSIONS.

1. The values of the maximum unit load in these tests vary over a considerable range for any given slenderness ratio and manner of fastening the angles in the testing machine.

2. In most cases the specimen which sustained the greatest unit load for a given slenderness ratio and method of fastening suffered the least lateral deflection and the angle which bent most sustained the lowest unit load at failure, the deflection being measured at 4/9 of the theoretical maximum load.

3. For large slenderness ratios the average values are well represented by Euler's formula for long columns, calculated for different values of the end fixation factor.

4. The Kármán curves, recalculated for a yield point of 37,000 lbs./in.^{2} and modulus of elasticity of 30,000,000 lbs./in.^{2} represent the average results for small slenderness ratios for several methods of end fixation, except in the neighborhood of $l/r=80$ to 85, where the effect of eccentricity was greatest. The values of the end fixation factor are given in Table 13.

5. For angles with ends folded the column formulas considered do not represent the results found in this series of tests.

6. It is believed that these values of end fixation factor are of importance in the design of structures where the end conditions approximate those used in these tests, no matter what formula the designer prefers to use.

7. Eccentricity of loading produces a diminution of column strength. In these tests the greatest effect of eccentricity was observed in the neighborhood of a "free length" corresponding to $l/r=85$, which agrees with the results of Kármán's investigations.

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